NCTM Session: Model with Mathematics, Working Across Science and Math in High School

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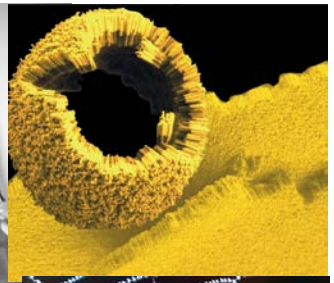
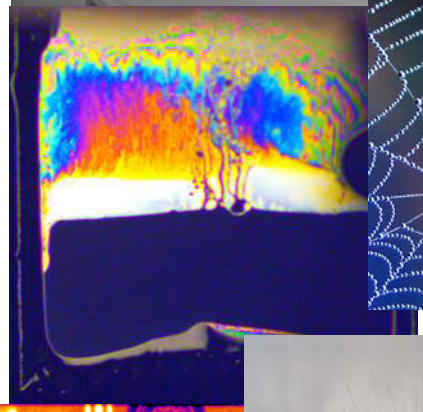
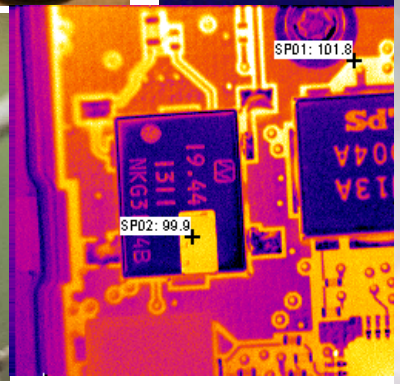





The
Microwaved
Peep
Experiment
Mom to 2 Posh Lil Divas


SP01: 101.8
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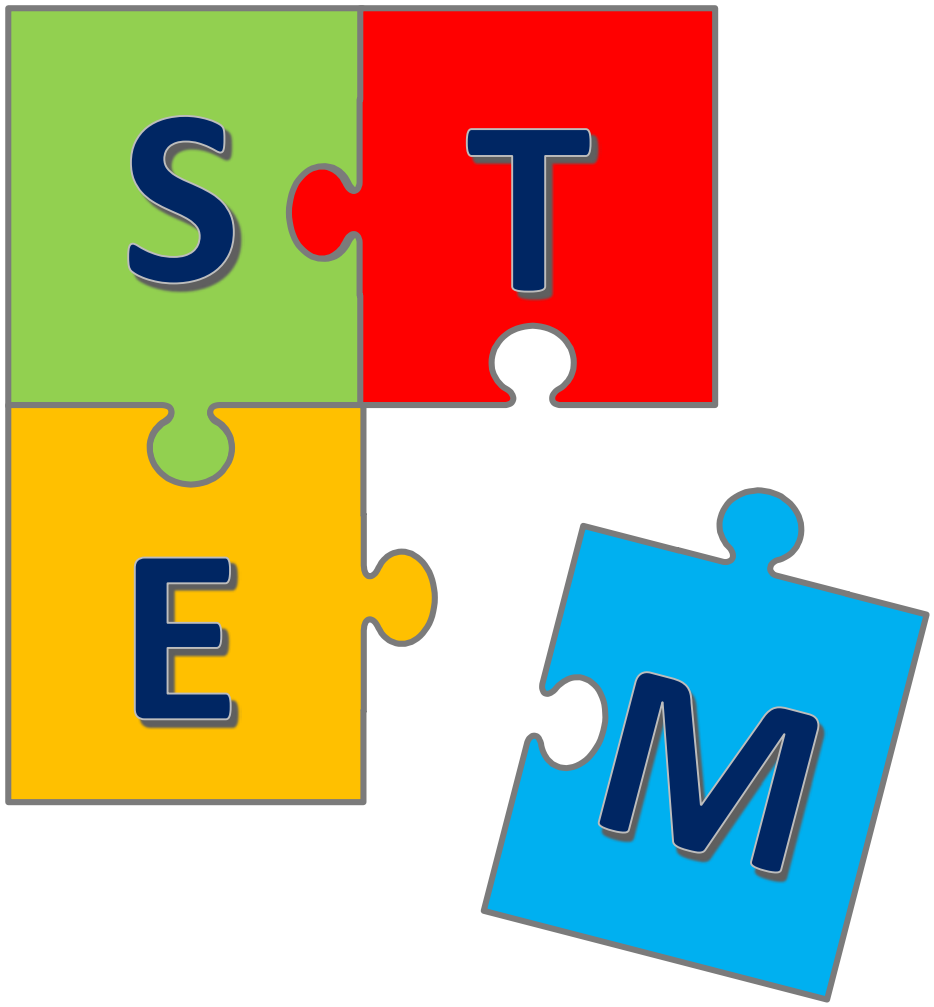
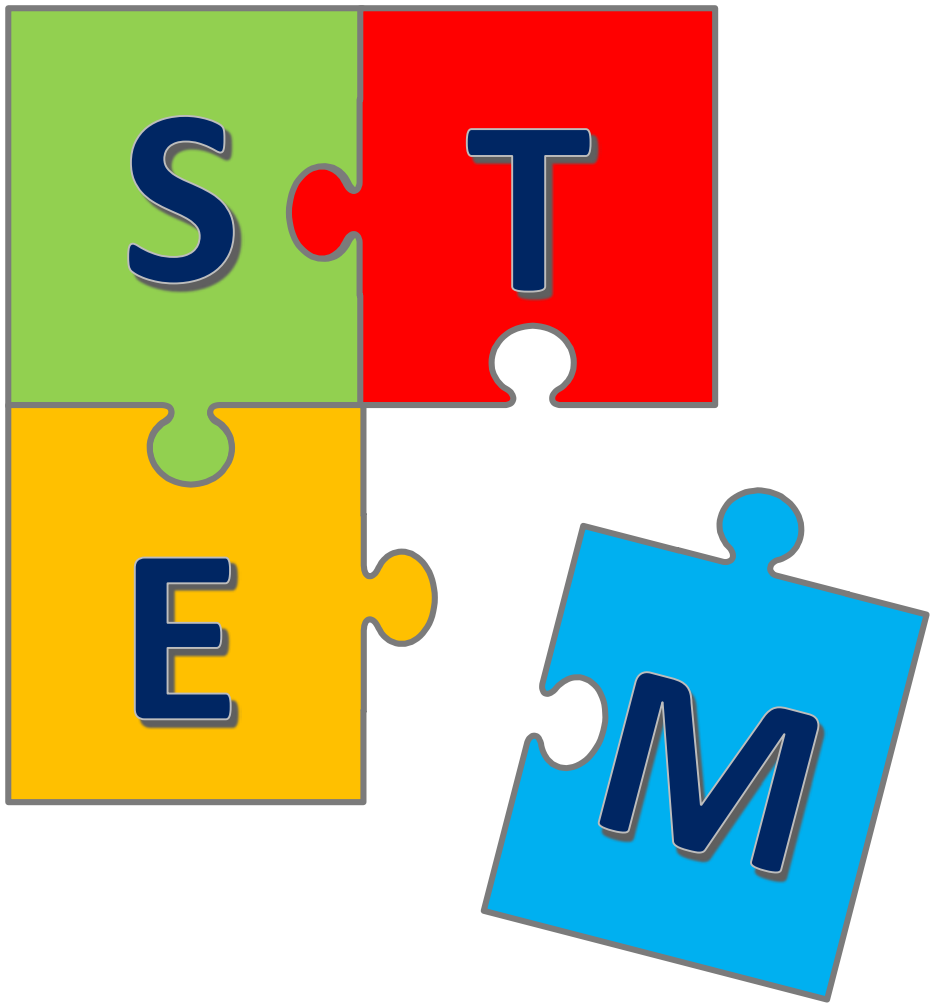
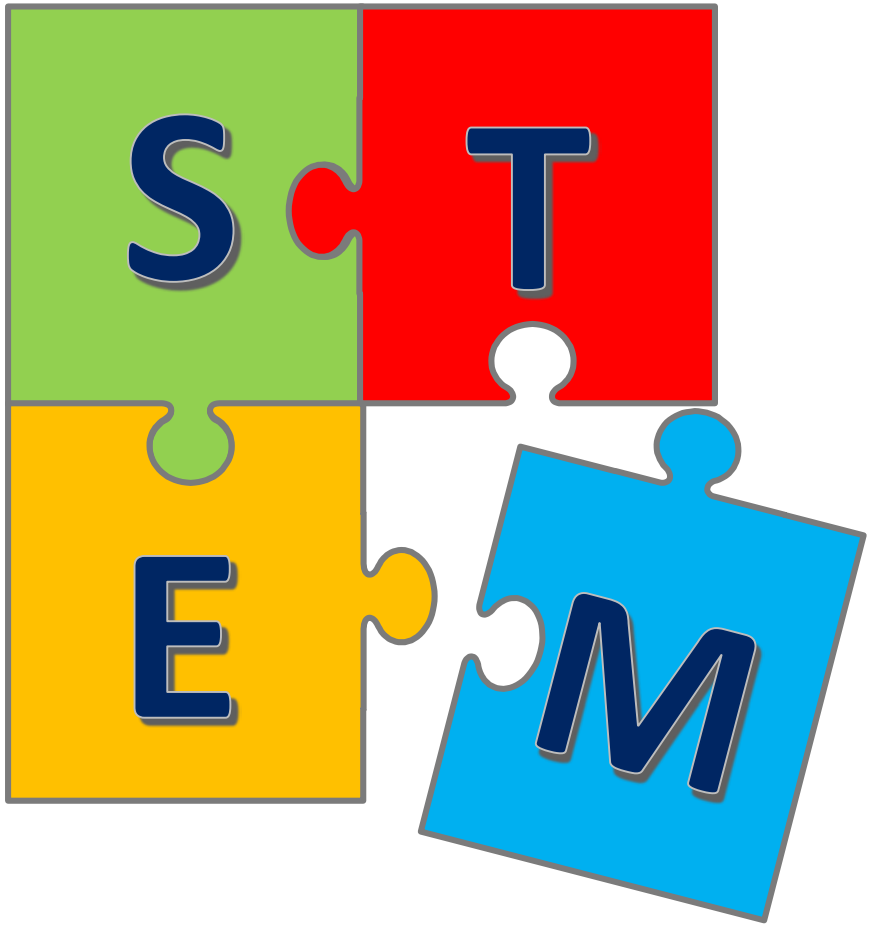


How does mathematics fit into the STEM puzzle? How do we design STEM activities so that students can experience the full power of mathematics as a tool for understanding the natural world?

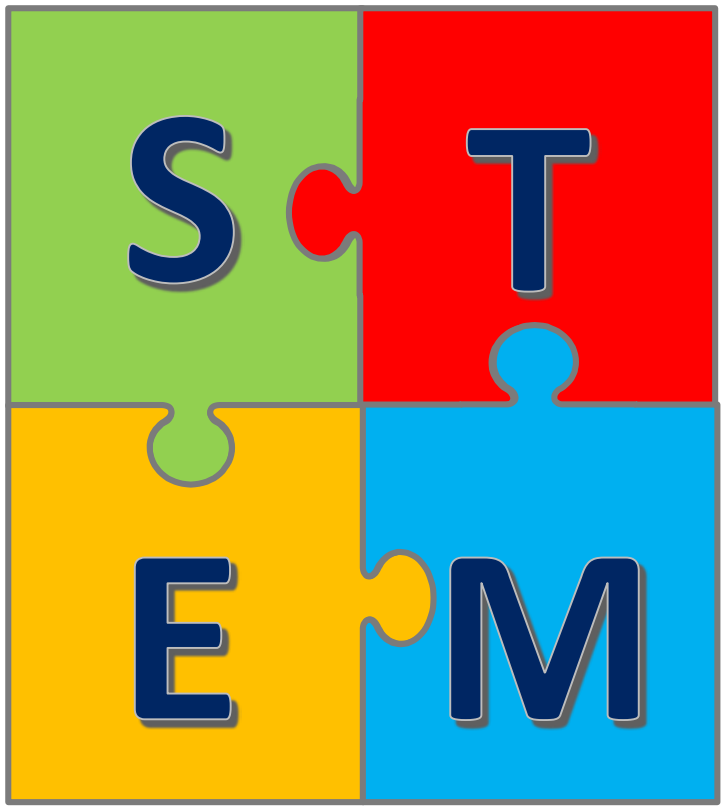


How is “M” incorporated into STEM activities in your school? How are math faculty involved in STEM? What are the challenges to both?

Challenge: A Vision for "M" in STEM



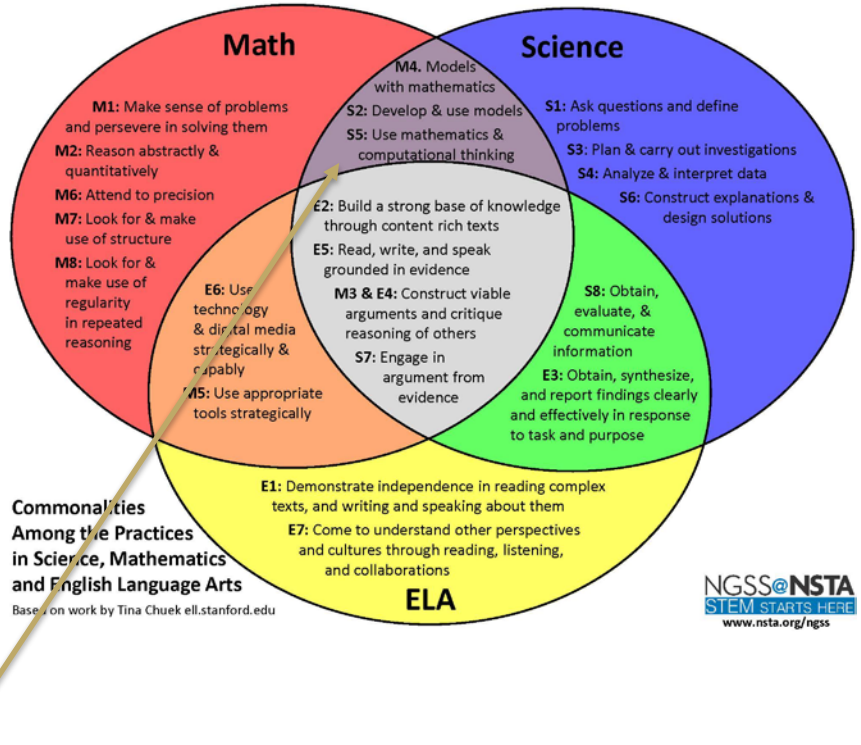
How is mathematics *integral* to STEM? How do we give students a *unified* view of the practices of science, mathematics, and engineering?

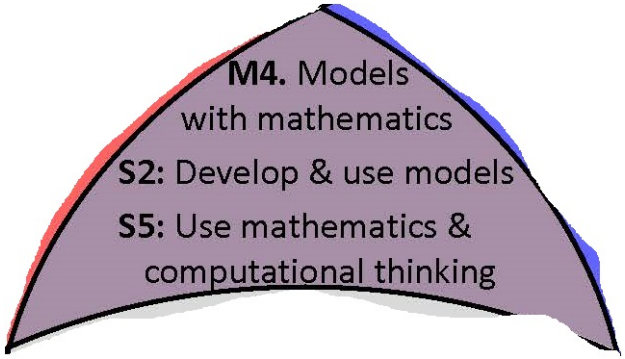


It's through the art of *mathematical modeling* that the "M" becomes an indispensable part of STEM.

COMMON CORE STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

NEXT GENERATION SCIENCE STANDARDS
For States, By States



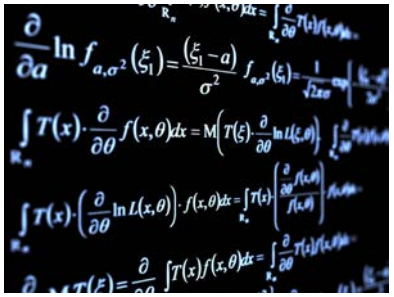


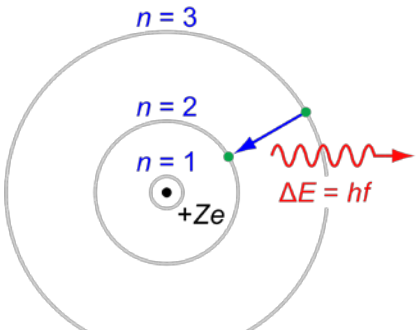
Models and modeling are a key common practice across science and mathematics.

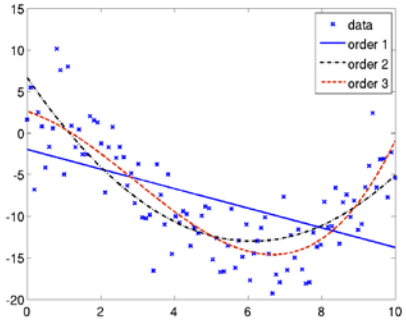
Goal #1 – Understand the answer to the question “What is mathematical modeling?”

Goal #2 – Experience mathematical modeling in a STEM activity.

What is a "*model*"?

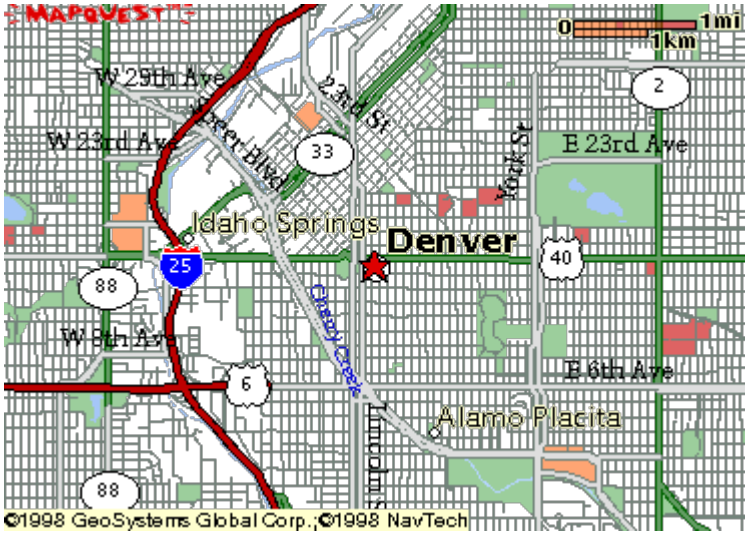






What is a "*model*"?

A *model* is a representation of a target that captures features or aspects of the target, but is not identical to the target.



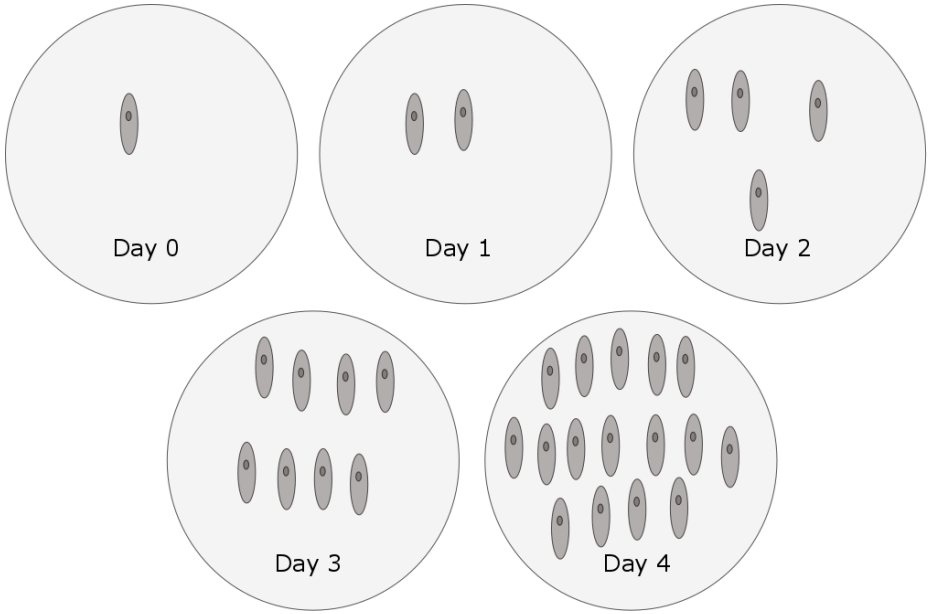
The notion of a *scientific model*

A *scientific model* is a model comprised of a set of ideas that describes a process, pattern, or phenomenon in the natural world.

Scientific Model – The raw egg contains a liquid that remains in motion even when the teacher's finger is placed upon the egg. When her finger is removed, this liquid pushes on the shell and returns the egg to motion. The hard boiled egg contains solid egg that fills the shell and when the motion of this egg is halted, the inside is halted as well.

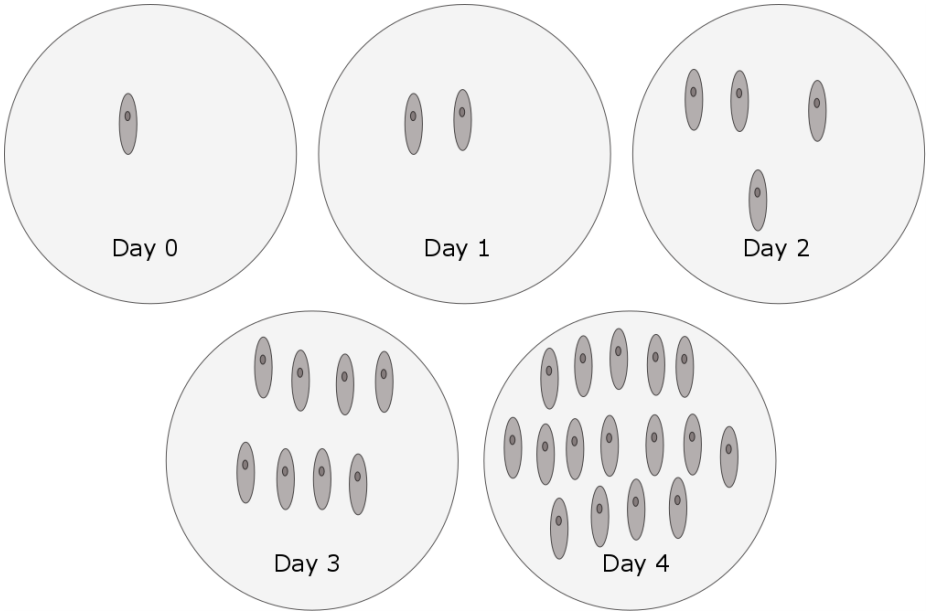
The notion of a *mathematical model*

A mathematical model is a *scientific model* where the ideas comprising the scientific model are expressed using mathematical objects.



Example: Simple population biology problem with bacteria.

The notion of a *mathematical model*



Scientific Model – The population of microorganisms is increasing daily through reproduction. Each microorganism present in the population reproduces once each day.

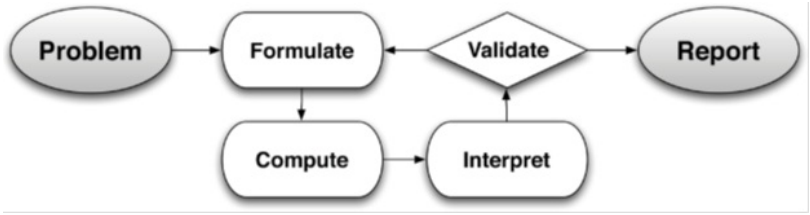
Mathematical Model -

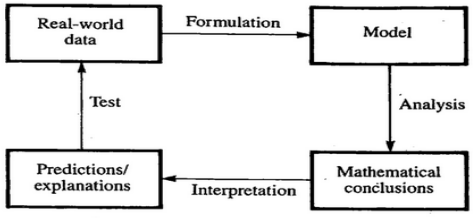
$$P(n+1) = 2P(n)$$

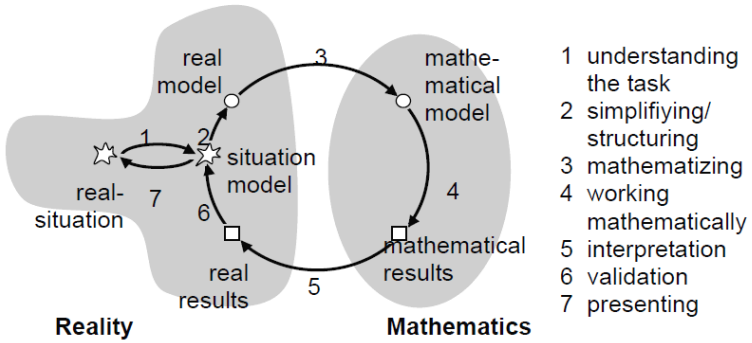
$$P(0) = 1$$

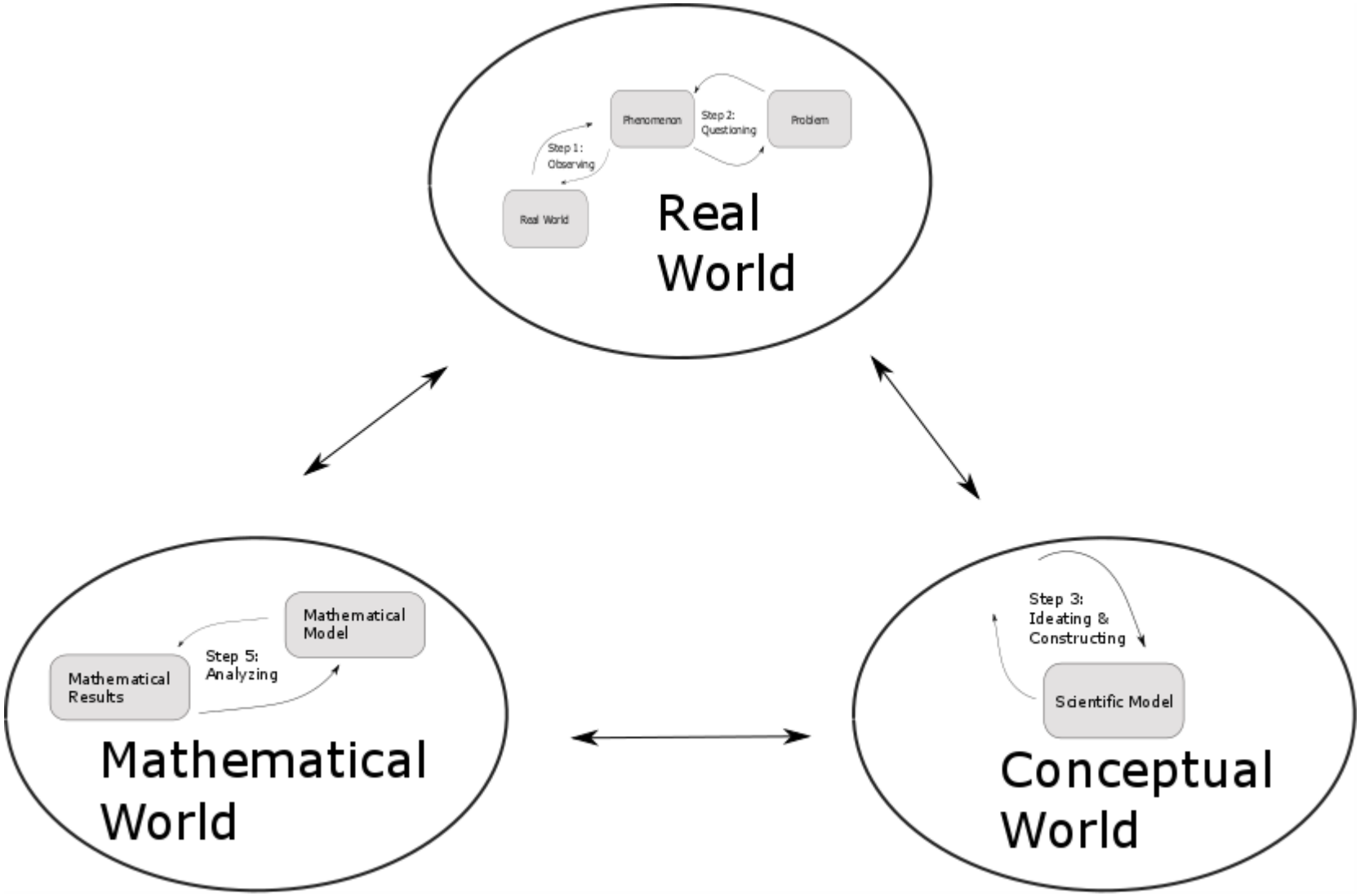
What is *mathematical modeling*?

Mathematical modeling is the process of constructing, analyzing, and revising mathematical models. We build mathematical models for the same reasons we build scientific models, that is, to *explain, predict*, and *control* processes, patterns, and phenomena in the real world.

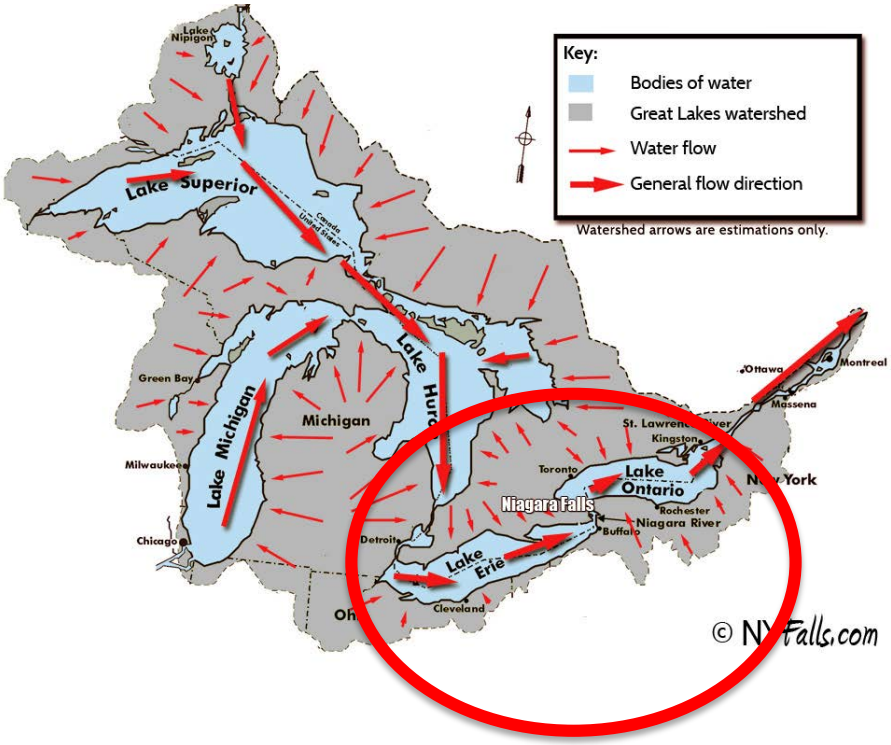






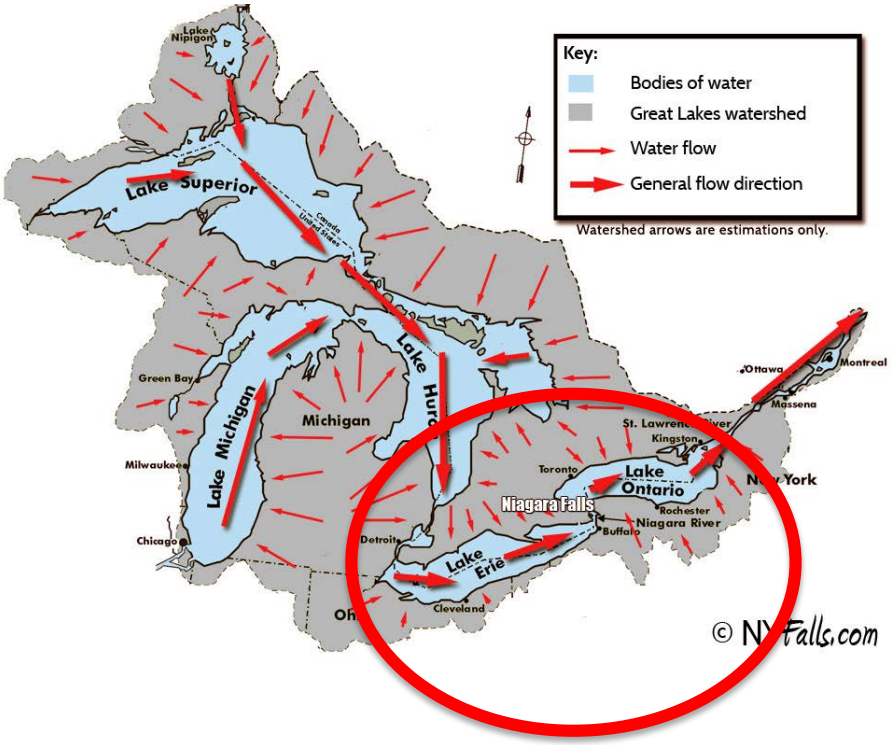

Phenomenon
Step 2: Questioning
Problem
Step 1: Observing
Real World
Real World
Mathematical Model
Step 5: Analyzing
Mathematical Results
Mathematical World
Step 3: Ideating & Constructing
Scientific Model
Conceptual World

What does this look like in a STEM activity?



Great Lakes Activity – This is a typically a full-day project that we do in professional development workshops. As a STEM activity, we use it to explore the science of pollution, engineering design, and mathematical modeling.

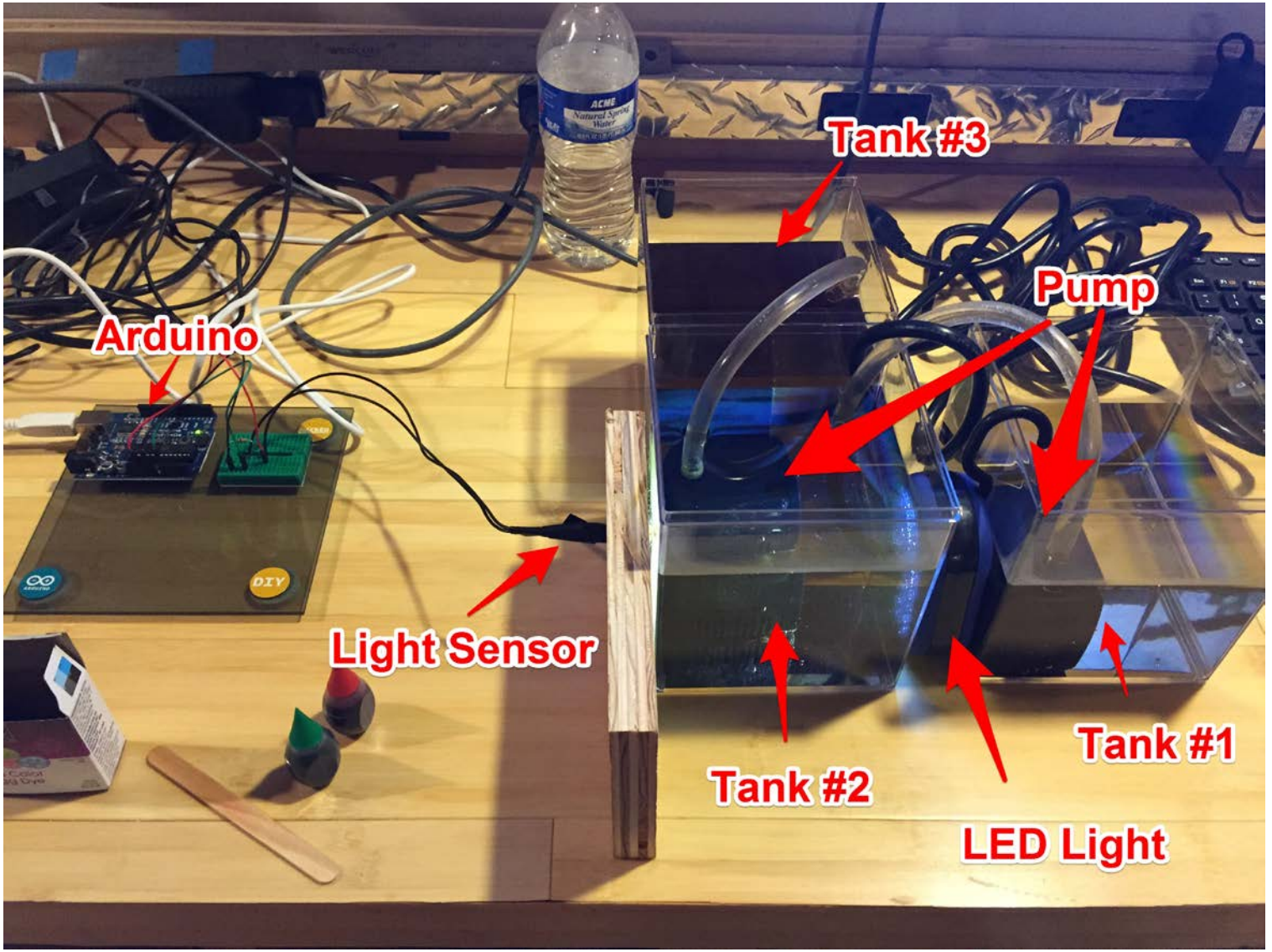
Today, we're just going to focus on a small piece of the larger activity. Our goal here is for you to work with a team on moving from the Real World to the Conceptual World to the Mathematical World.

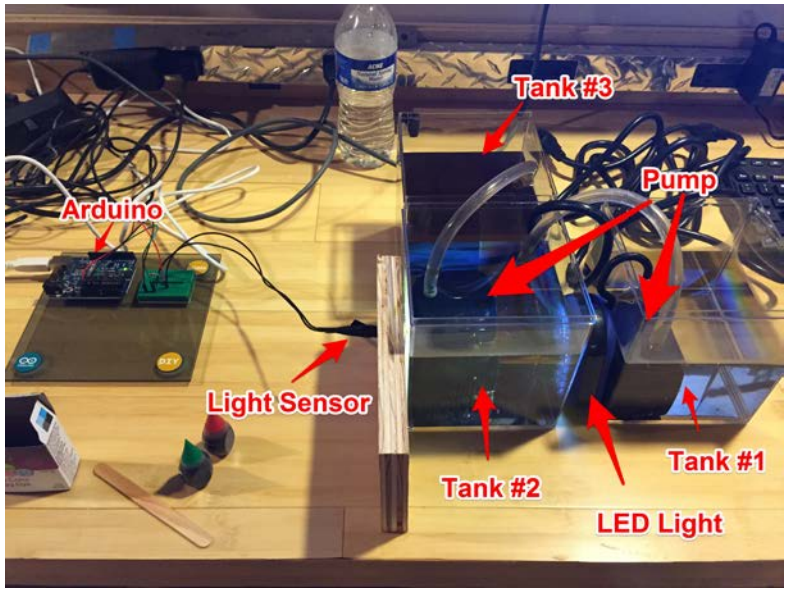


Challenge: Design and build a simple experimental system to explore how pollution levels change in a body of water when there is an influx of clean water and an outflow of contaminated water. Build a mathematical model of your simplified system to gain insight into the real Great Lakes problem.

Note – Involves science, engineering design, and mathematical modeling.

Solution: Build a simple three tank system with constant flow rates between the tanks. Insert "pollution" into the middle tank and monitor pollution level in this tank.

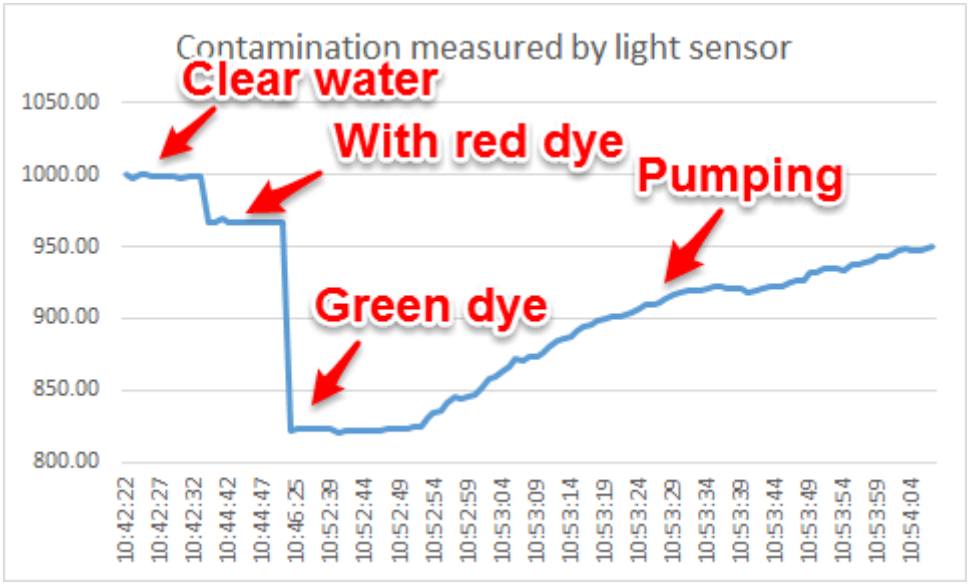

Tank #3
Pump
Arduino
Light Sensor
Tank #1
Tank #2
LED Light

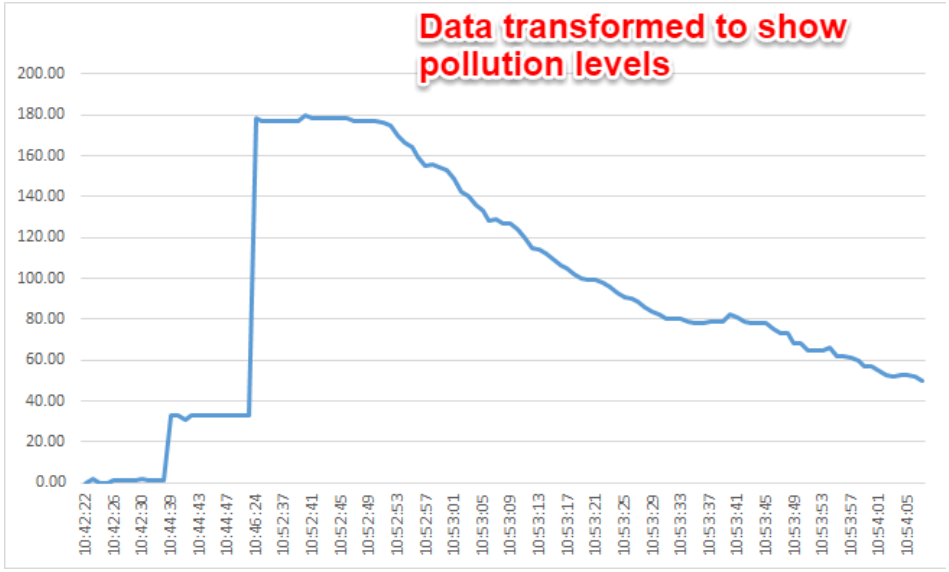


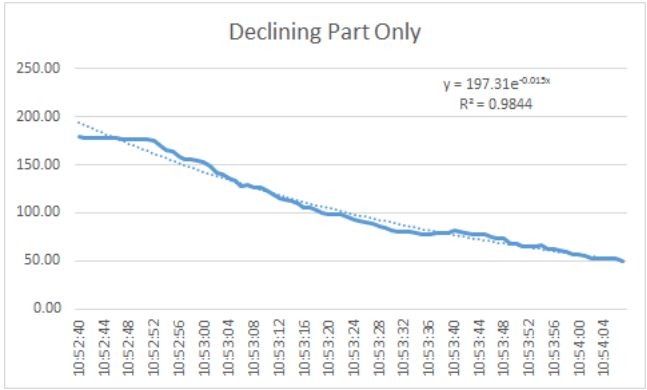
Task for Today – Build a mathematical model of the pollution level in the middle tank. Try first clearly laying out your scientific model of the process. Can you predict what the curve of pollution vs. time will look like from your model?

Take 20 minutes at your table for this activity and be prepared to share at the end of 20 minutes.

Clean water → Contaminated water → Outflow tank

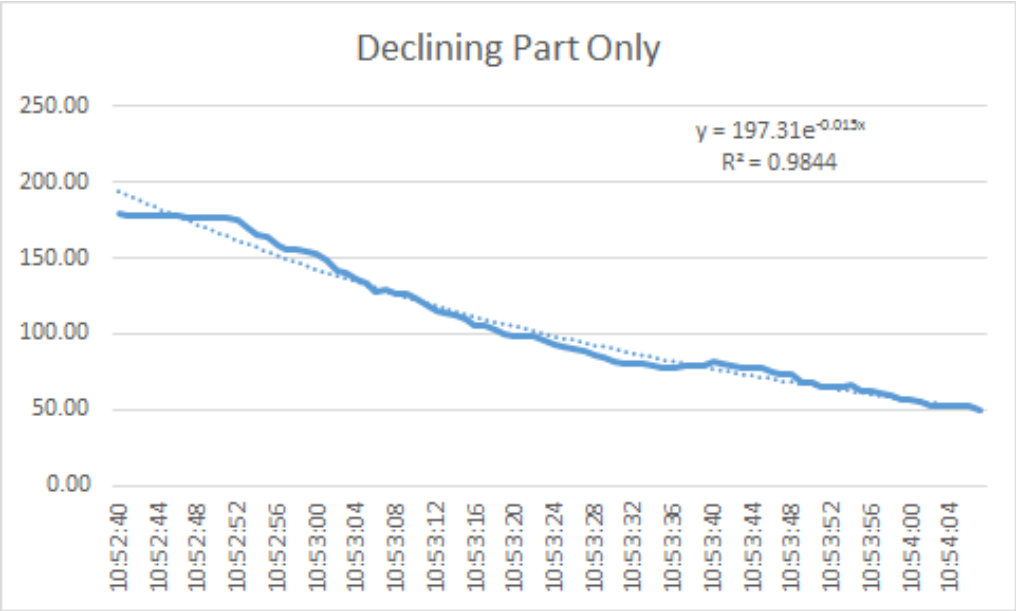

Contamination measured by light sensor
Clear water
With red dye
Pumping
Green dye
1050.00
1000.00
950.00
900.00
850.00
800.00
10:42:22
10:42:27
10:42:32
10:44:42
10:44:47
10:46:25
10:52:39
10:52:44
10:52:49
10:52:54
10:52:59
10:53:04
10:53:09
10:53:14
10:53:19
10:53:24
10:53:29
10:53:34
10:53:39
10:53:44
10:53:49
10:53:54
10:53:59
10:54:04


Data transformed to show pollution levels
200.00
180.00
160.00
140.00
120.00
100.00
80.00
60.00
40.00
20.00
0.00
10:42:22
10:42:26
10:42:30
10:44:39
10:44:43
10:44:47
10:46:24
10:52:37
10:52:41
10:52:45
10:52:49
10:52:53
10:52:57
10:53:01
10:53:05
10:53:09
10:53:13
10:53:17
10:53:21
10:53:25
10:53:29
10:53:33
10:53:37
10:53:41
10:53:45
10:53:49
10:53:53
10:53:57
10:54:01
10:54:05


Declining Part Only
y = 197.31e-0.015x
R² = 0.9844
250.00
200.00
150.00
100.00
50.00
0.00
10:52:40
10:52:44
10:52:48
10:52:52
10:52:56
10:53:00
10:53:04
10:53:08
10:53:12
10:53:16
10:53:20
10:53:24
10:53:28
10:53:32
10:53:36
10:53:40
10:53:44
10:53:48
10:53:52
10:53:56
10:54:00
10:54:04

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.

Eugene Wigner



Scientific Model –

Volume of water remains constant in the contaminated tank. The pollution level changes because of clean water pumped in at a constant rate and contaminated water pumped out at a constant rate.

Mathematical Model -

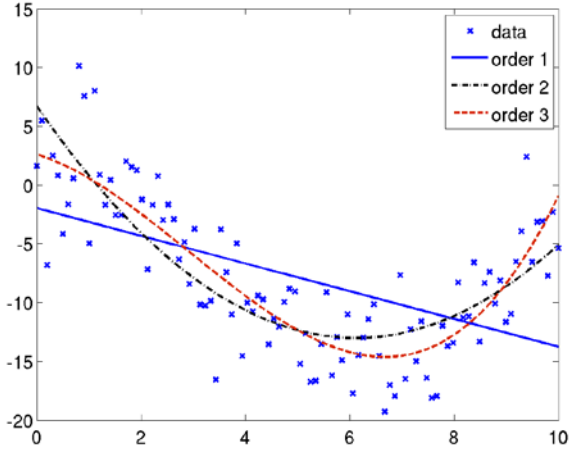
$$C(t+dt)V = C(t)V - C(t)Rdt$$

$$C(0) = A$$

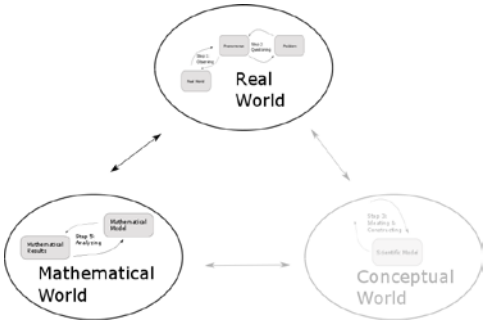
Analysis of the mathematical model reveals that the decay rate of pollution is simply R/V. This allows for comparison with both the simple experimental system and tells us what is crucial to know about the real Great Lakes system.

But, WAIT! I thought mathematical modeling was about curve fitting…

In *descriptive modeling*, a model simply describes the phenomena or summarizes them in a compact form… *Analytic modeling* seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based…



In *descriptive modeling* of the hanging chain, we might fit a curve to a set of measurements to approximate the shape.



In *analytic modeling*, we first build a scientific model, "the chain hangs in such a way that energy is minimized" and express that idea mathematically to *predict* the shape that we'll observe.