The Question Formulation Technique: A Tool for Teaching Mathematical Modeling

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Abstract

Mathematical modeling may be characterized as the art of asking good questions about the real world and then using mathematics to formulate models toward understanding the observed phenomena. Yet there is a negative correlation between time spent in school and the percentage of children actively using the skill of asking good questions (Right Question Institute 2015). How then can we foster curiosity in our students and support them to ask good questions about the world around them? We have found that the Question Formulation Technique (the QFT; Rothstein & Santana 2013), a step-by-step procedure designed to facilitate the asking of many questions, is a useful tool for teaching modeling. In this chapter, we begin by discussing the challenges of teaching mathematical modeling and the characteristics a capable modeler must possess. We describe the QFT and provide examples of how it can be used to facilitate the teaching of the art of modeling and how it has been used by secondary classroom teachers to teach mathematical modeling.

The world is filled with fascinating phenomena. Whether we look at the biological world and observe the pattern of stripes on a zebra, the social world and wonder how many ways we are connected to others on earth, or the natural world and wonder about convection patterns in our morning cup of coffee, we can observe endless phenomena that beg for explanation. Mathematics and, in particular, mathematical modeling, provides a powerful tool for investigating a wide range of such phenomena. We define mathematical modeling as the art or process of constructing a mathematical representation of reality that captures, simulates, or represents selected features or behaviors of that aspect of reality being modeled. The most distinguishing feature of using mathematics in this way is the cyclical process of mathematical modeling – observing something in the real world, questioning, mathematizing the phenomena, and then comparing a model of it to reality (see Figure 1).

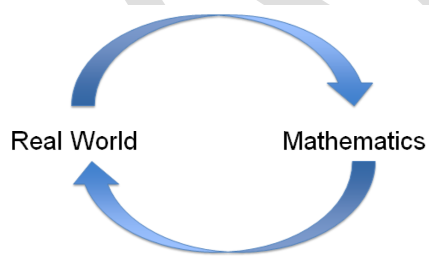

Figure 1: The cycle of connecting the real world and mathematics

From an educational point of view, mathematical modeling differs from typical problem solving in that genuine modeling problems tend to be ill-defined, open-ended with no one correct answer, and require assumptions to be made in order to make progress on their solution. In addition, decisions must be made about which factors to attend to and which to ignore. Creativity is necessary. Another way to characterize mathematical modeling is as the art of asking good questions about the real world and then drawing on mathematics to formulate models that can be used to understand the observed phenomena. Questions are inspired by real world phenomena, and questions must be asked throughout the modeling process.

In this chapter, we introduce the reader to the Question Formulation Technique, and we demonstrate how to harness its power in the teaching and learning of mathematical modeling. The architects of this technique, which was developed outside of mathematics education, argued that formulating one's own questions is the single most essential skill for learning. We have found that the Question Formulation Technique (the QFT; Rothstein & Santana 2013), a step-by-step procedure designed to facilitate the asking of many questions and to develop the user's ability to ask powerful questions, is a useful tool for teaching modeling. Here, we first describe some central competencies necessary to become a successful modeler. We observe that these all rely on a foundation of asking good questions. We then describe the QFT, illustrate its use in the modeling classroom, and provide examples of how it has been used by secondary classroom teachers to teach mathematical modeling.

Mathematical Modelers' Central Competencies

From a practitioner's perspective, the central competencies that need to be developed in order to become a successful mathematical modeler include: identifying factors, making assumptions, understanding relationships, translating relationships into mathematics, and choosing functions and other mathematical objects.

When faced with a new problem, a mathematical modeler must be able to identify what information is important and what information is not. That is, important factors must be identified. Mathematical modelers must be able to make assumptions that simplify the task of building a model and that lead to the construction of a *tractable model* (i.e., a model that makes use of the mathematics that the modeler has access to). Mathematical modelers need to understand how different elements of a problem or system are related. They need the ability to identify these relationships and make them visible. Mathematical modelers must be able to translate the important factors in a problem and the relationships between them into mathematical terms. Finally, they need to draw from a range of mathematical sub-fields. They know how and when to choose appropriate mathematics to capture key features of a system.

All of these competencies rest on the ability to ask and answer one's own questions. The approach that we are promoting here, supporting the mathematical modeler to ask their own questions, lies in stark contrast to the typical approach that authors of modeling textbooks tend to take. In particular, one common approach is for textbook authors to provide a set of questions that students should be asking about a real world phenomenon. Questions include, for example: What do we know? What do we want to know? What can we assume? (Dynn 2004). However, from 20 years of experience with teaching mathematical modeling at the undergraduate and graduate levels, the first author has found that this approach does not work. In line with the findings of Rothstein and Santana (2013), telling students what questions to ask is actually counterproductive. Instead, students must be challenged to ask and answer their own questions as they work through the mathematical modeling cycle.

Rothstein and Santana (2013) further argue that the skill of posing good questions must be *taught.* This is important because research suggests that the percentage of students who actively use this skill decreases with age (Right Question Institute, 2015). As Figure 2 shows, children typically reach their question-asking peak around the age of four, and question-asking dramatically decreases once students enter the classroom. The Question Formulation Technique, described in a later section, supports the challenging task of teaching mathematical modeling.

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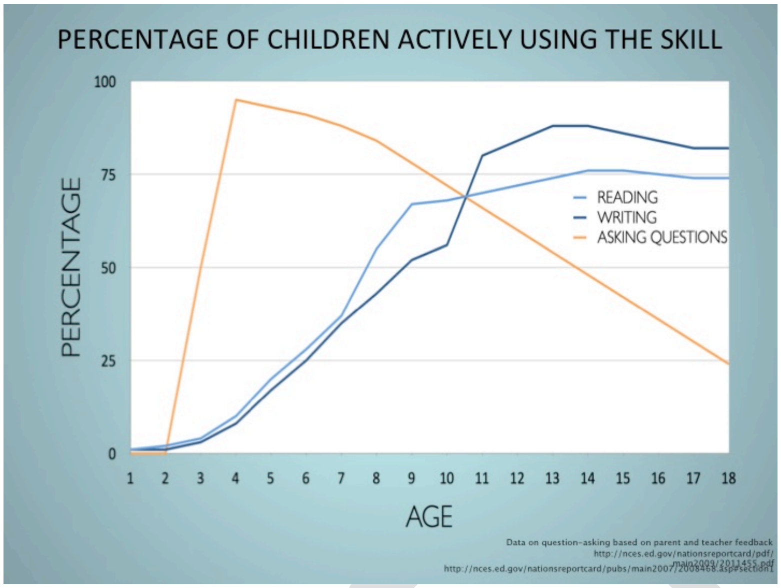

Figure 2: Question-asking by age

What is So Challenging about Teaching Mathematical Modeling?

Because mathematical modeling cuts across all strands of mathematics, determining how and where to integrate it into the curriculum can be complicated. In the CCSSM high school standards (NGA & CCSSO 2010), mathematical modeling is the only conceptual category that does not specify particular standards broken down into specific clusters and domains. Instead, modeling is supposed to be taught through the other content standards. Additionally, in the Modeling Progressions document, the authors stated:

> One of the eight mathematical practice standards – MP4 Model with mathematics – focuses on modeling and modeling draws on and develops all eight. This helps explain why modeling with mathematics and statistics is challenging. It is a capstone experience, the proof of the pudding. To embody this, students must complete a capstone experience in modeling. (Common Core Standards Writing Team, 2013, p. 8)

Yet neither the CCSSM document nor the companion Progressions document provides guidance to teachers as to what kinds of capstone experiences might help them address the mathematical modeling standards. Consequently, classroom teachers are left to consider questions such as: What counts as a modeling task? and How do I teach modeling through the other standards? This is a challenge since it has been noted that traditional textbooks, including those in problem-based curricula, rarely support genuine mathematical modeling (Verschaffel, Greer, & de Corte 2000).

This claim has been partially validated by a recent curriculum analysis of a purported "Common Core-aligned" textbook series. More specifically, in his analysis of 83 tasks labeled "Modeling" in two textbooks (Algebra 1 and Geometry), Meyer (2015) found that the modeling tasks were mainly focused on performing operations and interpreting results. Tasks that asked students to identify essential variables or compare the conclusions drawn from their mathematical models with real-world results were rare. Thus, teachers will also need support for identifying or creating modeling tasks and then supporting students to explore them.

Perhaps the most challenging aspect of teaching mathematical modeling is helping students overcome the "Where should I start?" barrier. Even in a typical undergraduate mathematical modeling class or lesson, the instructor walks students through the modeling process in the context of a particular problem. The problem is usually chosen, not to illustrate the modeling process, as much as it is chosen to highlight the use of a particular type of mathematics in context. The majority of the time is then spent on the mathematical analysis stage of the process, and at the end, students are given prompts that ask them to fill in details of the analysis or perhaps slightly extend the model. From experience we know that when students are then challenged to "start from scratch" and completely model and analyze an unfamiliar situation, the "Where should I start?" refrain becomes common.

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We argue that the key to overcoming this challenge at all levels is to think critically about the modeling cycle and the ways in which the typical teaching approach fails to develop the key competencies that students need right at the start of the modeling cycle. For concreteness, let us use the mathematical modeling cycle found in CCSSM to illustrate (see Figure 2).

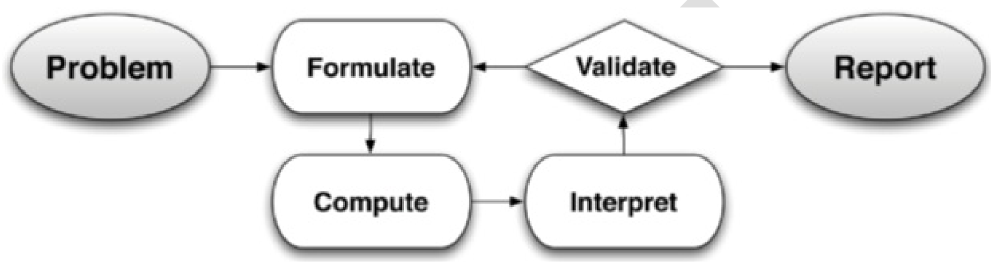

Figure 2: The Common Core mathematical modeling cycle (NGA & CCSSO, 2010)

We note that this cycle, as do most, hides the complexity of the skills needed in first, defining or getting to the "problem" in the beginning step, and second, in the arrow connecting the "Problem" to "Formulate" steps of the process. We also note that the skills needed to get to the problem or move from the problem to the formulation are generally speaking not mathematical skills, but rather are "soft skills" less easy to define and perhaps less comfortable for a typical mathematics instructor to teach. We also argue that the most critical of these skills is simply the art or habit of asking questions. If, as mentioned above, mathematical modeling is truly the art of asking good questions, it is precisely at the point of choosing the "Problem" and choosing how to move from "Problem" to "Formulate" that the skill of asking questions is most important. Hence, when confronted with an unfamiliar situation, the student without this habit is reduced to asking the single question, "Where do I start?"

Next, we introduce the "Question Formulation Technique," a powerful pedagogical tool developed to rebuild the skill of asking questions in students. We briefly explain the technique and then illustrate its use in re-envisioning the mathematical modeling classroom.

What is the Question Formulation Technique?

Rothstein and Santana (2013), authors of *Make Just One Change*, argued that: (a) all students should learn how to formulate their own questions, and (2) all teachers can easily teach this skill as part of their regular practice (p. 1). To support these goals, they proposed the Question Formulation Technique (QFT), a simple, but rigorous, step-by-step process designed to help students produce questions, improve the questions, and then plan how to use their questions. This process supports three thinking abilities in one process: divergent, convergent, and metacognitive thinking. The QFT has been developed, tested, and improved over the last two decades (Rothstein and Santana 2013). The QFT protocol has seven components, which are summarized in Table 1.

Table 1: The Question Formulation Technique Explained

Component	**Description**
The Question Focus	The Question Focus (QFocus) is usually developed by the teacher and serves as the jumping off point for student questions. It should be brief and provoke or stimulate new lines of thinking. The QFocus should not be in the form of a question.
The Rules for Producing Questions	The rules for producing questions must be discussed in advance and understood by the students. The rules for producing questions are: • Ask as many questions as you can • Do not stop to answer, judge, or discuss the questions • Write down every question *exactly* as it is stated • Change any statement into a question.

	Post or distribute the Rules for Producing Questions.
Producing Questions	After discussing the rules, divide students into groups of 3-5 students. Ask each group to identify a note-taker. Distribute or reveal the QFocus. Students should produce as many questions as possible in the allotted time (start with 10 minutes; decrease time as students master the technique). The rules should be followed, and the questions should be numbered. The note-taker should contribute questions.
Categorizing Questions	Questions should be categorized as closed or open-ended questions: - **Closed-ended Questions** can be answered with a "yes" or "no" or with a one-word answer. - **Open-ended Questions** require more explanation. Students should label the questions with a "C" or an "O" and practice changing closed questions to open questions and vice-versa.
Prioritizing Questions	Following criteria set by the teacher, students should prioritize their questions and choose three questions. Examples of criteria include questions that: most interest you, you consider to be the most important, will best help you solve a problem, or you need/want to answer first.
Sharing Questions & Next Steps	Students can be asked to share: • Their three priority questions • Their reasons for choosing the priority questions • The numbers of the priority questions in the sequence of the

	entire list. Students should use their questions to conduct research or write papers.
Reflection	Students should name what they have learned, how they learned it, and how they will use what they learned.

Through their years of experience with facilitating the QFT, Rothstein and Santana have accumulated advice for teachers attempting to use the QFT. We strongly recommend making use of the resources on the Right Questions Institute website (http://rightquestion.org/education/). First, some effort should be invested to learn how to create an effective question focus, but you can pick it up pretty quickly. Second, the authors suggest that you do not give examples to students, but rather make good use of the *Rules for Producing Questions.* Third, it is helpful to manage the groups by monitoring to ensure that they are staying on task and following the rules. Fourth, it is important to make reflection part of the process. Students should be asked to *think about their thinking and talk about it*. Last, because getting students to ask, rather than answer questions might seem strange to them, be sure to encourage them, giving plenty of positive feedback, especially to the quieter students (Rothstein and Santana 2013).

Using the Question Formulation Technique to Teach Mathematical Modeling

Here, we illustrate how the QFT can be used in teaching the art of mathematical modeling. The successful use of the QFT in this context depends on rethinking the typical "launch" of a modeling exercise where the phenomena to be studied is described and the key questions, ready to be mathematized, are posed by the instructor. Instead, the instructor must let go of any compunction to be the question poser, and, instead, use the phenomena of interest as the basis for designing an effective question focus. The QFocus should be as simple as possible, should spark curiosity, and should allow for a wide range of questions to be asked. As always with the QFT, the instructor must resist the temptation to over-determine the QFocus in order to lead students down a particular pathway.

As an example of how this may be done effectively, we consider a staple of mathematical modeling classrooms, namely the spread of infectious disease. Lending itself to relatively clean and interesting mathematical analyses and suitable for learners with a wide-variety of mathematical backgrounds, problems in epidemiology are indeed well-suited to a lesson in mathematical modeling. The typical approach to introducing such problems is to start with something along the lines of "Imagine we have a population of individuals, some of whom are infected, some of whom are susceptible to infection, and some of whom have recovered." We note that starting at this point presupposes that the student recognizes questions such as "How does the spread of a disease vary with time in a population?" as interesting, relevant, and mathematically accessible though, in fact, many students will fail to even recognize this as the question we are attempting to answer.

A QFT-based approach to the same problem may start with a simple prompt such as the one shown in Figure 3 or, alternatively, "Ebola is a disease that is spreading rapidly." In the case of the figure below, all the instructor should say is that this is a chart, produced by the World Health Organization that illustrates the 2014 Ebola outbreak in West Africa. With this prompt, the instructor then directs the students to begin asking questions.

The next crucial step for the instructor is the prioritization step, where the prioritization criteria have been carefully formulated in advance. An effective criterion in the modeling classroom is often "Which are the most important questions that can be answered using a mathematical model?" Note that this criteria is effective even if students do not yet truly grasp

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mathematical modeling at all as it draws their attention to *possible* connections between their questions and mathematics. When used in the classroom of even as many as sixty students working in groups of four and five, it is remarkable how at least one of each groups top questions is some variant of “How does the spread of a disease vary with time in a population?” Through a whole-class discussion it is easy to draw their attention to this question and have the class agree to focus on this question and its refinement.

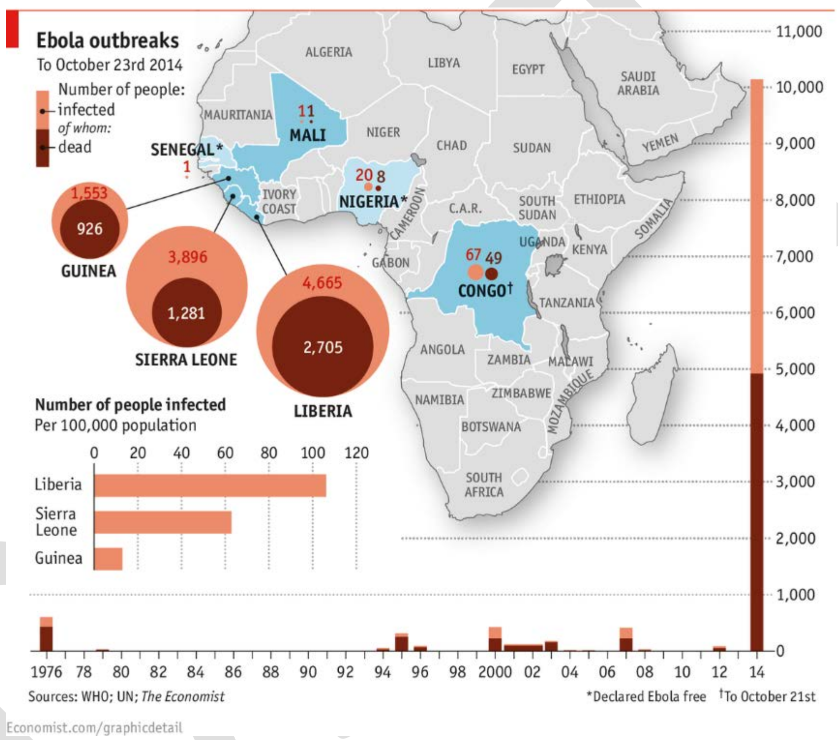

Figure 3: The spread of Ebola

The act of refining the question begins to spark the development of the central competencies mentioned above. Now in the habit of and freed to ask questions, students almost automatically begin to say “What factors are important in how a disease spreads?” and “What if we assume that you either have the disease or you don’t and once you have it you always have it?” Such questions are at the heart of the competencies of identifying factors, making assumptions, understanding relationships, translating relationships into mathematics, and choosing functions and other mathematical objects. How to build on this reignited curiosity is now at the discretion of the instructor and many paths are possible. The instructor may choose to have students develop further questions or identify those questions that must be answered to answer this one central question. The instructor may have students begin to answer the question in small groups or do further research for homework. Or, the instructor may now take them down the path of their favorite epidemiological model, but this time, more certain that students want to follow and have a better sense of why they are moving down a particular pathway.

Students' and Teachers' Reactions to Using the QFT from Classroom

We have used the QFT with secondary mathematics teachers in professional development sessions designed to improve their mathematical knowledge for teaching modeling. Teachers responded quite positively to learning the QFT. At the end of a week-long summer professional development seminar, comments such as "I really liked the QFT model and the formalization of questioning" and "The QFT was great - I will be using it next year" were common.

We heard from many teachers throughout the school year who successfully made use of the QFT. Here we highlight the work of first year teacher Carla and her mentor, veteran teacher, Kathy. Carla and Kathy teach in the same school and planned several QFT lessons together. They wanted to use the structure of the QFT to support student participation in mathematical modeling lessons. They aimed to decrease teacher dependence and increase student collaboration. They also sought to re-inspire curiosity in their students and support them in engaging with the reasoning of others. When Carla and Kathy use the QFT, they allow their students to write their questions with dry-erase markers on their desks (see Figures 4 and 5). These teachers used the QFT throughout the school year. One way that the teachers connected to the students' lived experiences, was through the following QFocus: The number of hours of

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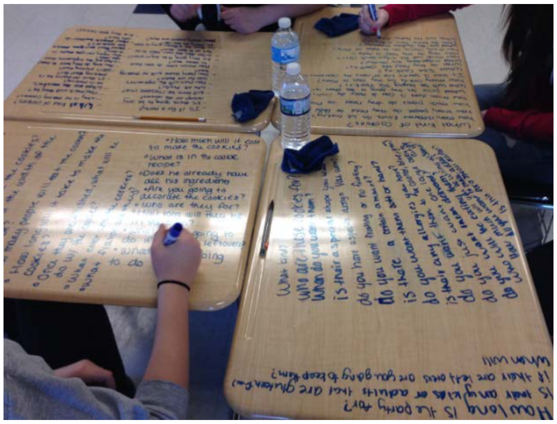
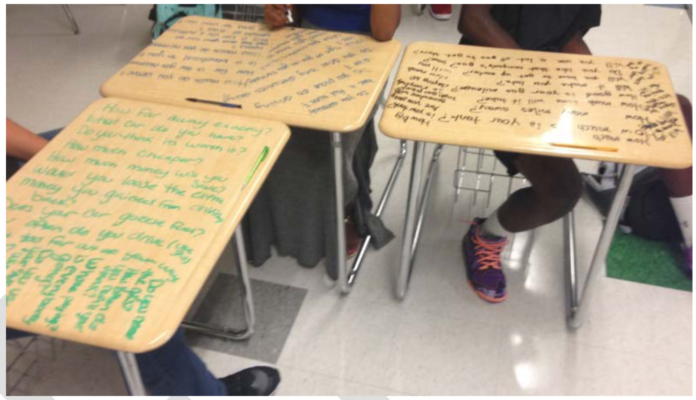
daylight in OurTown [blinded for review] varies throughout the year. Carla and Kathy were surprised and delighted to be able to quickly support students to develop mathematical models of the situation that were genuinely their own and not all the same.





Figures 4 and 5: Students practice the Question Formulation Technique on their desks.

In reflecting on their use of the QFT, Carla and Kathy had wonderful things to say. For example, Carla wrote in an email: “I honestly think that the QFT has done great things for my classes. They have actually *asked* for me to prove new theorems we have learned so that they understand *why* they work.” Both Carla and Kathy presented at a subsequent professional development session and reported that teaching the QFT and mathematical modeling changed the culture of their classrooms by re-igniting curiosity. They said that students now ask “why” even when they are not working on the QFT. Also students seem more concerned with asking whether or not their answers make sense than they were previously, and students’ confidence seemed to improve when they were able to see that they can figure out challenging problems.

Conclusion

The QFT is a strategy that has been used throughout the world in a variety of settings. The results of its use are impressive. Teachers who have used the QFT reported that the process leads to students who take far more ownership of their learning than they have ever seen.

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Students have also reported positive effects from its use. We have used the QFT with teachers, and the teachers we worked with tried it out on their students. In both cases, the QFT served as a useful tool for teaching mathematical modeling as well as a formative assessment tool.

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